Floor and Ceiling Functions

Note Title 26/10/2006

Type : integer
real
boolean
array of integer

"Cast" conversion from one type to another

Summary

Convention : x, y, z real nos.
i, j, k, m, n integer nos.

Inequalities :

$[x < y \equiv x + z < y + z]$

$[x \leqslant y \equiv x + z \leqslant y + z]$

$[x < y \equiv x \times c < y \times c]$ for all c such that $0 < c$.

$[x \leqslant y \equiv x \times c \leqslant y \times c]$ for all c such that $0 < c$.

$[m < n \equiv m + 1 \leqslant n]$ NB. integer m, n.

Definition For all real x, $\lceil x \rceil$ is an integer such that, for all integers m,

$$\lceil x \rceil \leqslant m \quad \equiv \quad x \leqslant m \quad .$$

(Java-like cast notation:

$$(\text{ceil})\, x \leqslant_{\text{Int}} m \quad \equiv \quad x \leqslant_{\text{Real}} (\text{real})\, m \quad .)$$

$$\lceil x \rceil \leqslant m \quad \equiv \quad x \leqslant m \quad .$$

Consequences

(Truthify rhs) $\quad [\quad \lceil m \rceil \leqslant m \quad]$ NB. integer m

(Truthify lhs) $\quad [\quad x \leqslant \lceil x \rceil \quad]$

Hence (antisymmetry): $\quad [\quad \lceil m \rceil = m \quad]$

NB. integer m